

Chiral SU(3) dynamics and the quasibound K^-pp cluster

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Abstract The prototype of a \bar{K} nuclear cluster, K^-pp , has been investigated using effective $\bar{K}N$ potentials based on chiral SU(3) dynamics. Variational calculation shows a bound state solution with shallow binding energy $B(K^-pp) = 20 \pm 3$ MeV and broad mesonic decay width $\Gamma(\bar{K}NN \rightarrow \pi YN) = 40 - 70$ MeV. The $\bar{K}N(I=0)$ pair in the K^-pp system exhibits a similar structure as the $\Lambda(1405)$. We have also estimated the dispersive correction, p -wave $\bar{K}N$ interaction, and two-nucleon absorption width.

Keywords \bar{K} nuclei · chiral SU(3) dynamics · variational calculation

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1 Introduction

\bar{K} nuclei (nuclear systems with a bound anti-kaon) have recently become a hot topic in hadron and nuclear physics. With a phenomenological $\bar{K}N$ potential, it was suggested that the \bar{K} nuclei could exist as deeply bound states with small width [1]. Experiments performed in search for such states have so far been inconclusive [2]. An important prototype is the K^-pp system, the simplest \bar{K} -nuclear cluster. Recently this system has been studied using Faddeev [3, 4] and variational [5, 6] approaches with $\bar{K}N$ interactions constrained by scattering data and properties of the $\Lambda(1405)$.

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An essential ingredient to study \bar{K} nuclei is the $\bar{K}N$ interaction below threshold, which is only accessible through the subthreshold extrapolation of the amplitude adjusted to $\bar{K}N$ scattering data. Theoretical guidance is required for this extrapolation. Here we report on the study of a variational calculation of K^-pp system [7] using the effective $\bar{K}N$ interaction based on chiral SU(3) dynamics [8].

2 Formalism

The present variational investigation focuses on the K^-pp system with spin and parity $J^\pi = 0^-$ and isospin $(T, T_z) = (1/2, 1/2)$, where the parity assignment includes the intrinsic parity of the antikaon. Our model wave function for this K^-pp state, $|\Psi\rangle$, has two components:

$$|\Psi\rangle = \mathcal{N}^{-1} [|\Phi_+\rangle + C |\Phi_-\rangle],$$

$$|\Phi_+\rangle \equiv \Phi_+(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_K) |S_N = 0\rangle \times \left| \left[[NN]_{T_N=1} \bar{K} \right]_{T=1/2, T_z=1/2} \right\rangle, \quad (1)$$

$$|\Phi_-\rangle \equiv \Phi_-(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_K) |S_N = 0\rangle \times \left| \left[[NN]_{T_N=0} \bar{K} \right]_{T=1/2, T_z=1/2} \right\rangle, \quad (2)$$

where \mathcal{N}^{-1} is a normalization factor. The first, second and third terms in Eqs. (1) and (2) correspond to the spatial wave function, the spin wave function of the two nucleons (assuming $S_N = 0$), and the isospin wave function of the total system, respectively. We consider two different isospin states of the two nucleons ($T_N = 1$ in $|\Phi_+\rangle$ and $T_N = 0$ in $|\Phi_-\rangle$), while the $\bar{K}NN$ system in both cases has total isospin and third component $(T, T_z) = (1/2, 1/2)$. The dominant contribution is the $T_N = 1$ component corresponding to the leading K^-pp configuration. The mixing with the $T_N = 0$ component is caused by the difference between the $\bar{K}N$ interactions in $I = 0$ and $I = 1$. The spatial part of the wave functions are products of single particle wave packets and two-particle correlation functions. The NN correlation function permits an adequate treatment of a realistic NN potential with its strong short-range repulsion. The parameters in the model wave function are determined by minimization of the energy.

The Hamiltonian used in the present study is of the form

$$\hat{H} = \hat{T} + \hat{V}_{NN} + \text{Re } \hat{V}_{\bar{K}N} - \hat{T}_{CM},$$

where $\hat{V}_{NN}(\hat{V}_{\bar{K}N})$ stands for the $NN(\bar{K}N)$ interaction. Here \hat{T} is the total kinetic energy. The energy of the center-of-mass motion, \hat{T}_{CM} , is subtracted. As a realistic nucleon-nucleon interaction \hat{V}_{NN} we choose the Argonne v18 potential (Av18) [9]. We employ the central, L^2 and spin-spin parts of the Av18 potential for the singlet-even (1E) and singlet-odd (1O) channel, since the total spin of the two nucleons is restricted to zero in our model. We use the $\bar{K}N$ interaction $\hat{V}_{\bar{K}N}$ derived from chiral SU(3) dynamics [8]. This complex and energy-dependent interaction is parametrized by a Gaussian spatial distribution:

$$\hat{V}_{\bar{K}N} = \hat{v}(\bar{K}N_1) + \hat{v}(\bar{K}N_2)$$

$$\hat{v}(\bar{K}N) = \sum_{I=0,1} \hat{P}_I(\bar{K}N) \times v_{\bar{K}N}^I(\sqrt{s}) \exp \left[-(\mathbf{r}_{\bar{K}} - \mathbf{r}_N)^2 / a_s^2 \right],$$

where $\hat{P}_I(\bar{K}N)$ is the isospin projection operator for the $\bar{K}N$ pair. The interaction strength $v_{\bar{K}N}^I(\sqrt{s})$ is a function of the center-of-mass energy variable \sqrt{s} of the $\bar{K}N$

subsystem. The strength $v_I^{\bar{K}N,S}(\sqrt{s})$ and the range parameter a_s are systematically determined within the chiral coupled-channel approach.

The energy dependence of the $\bar{K}N$ interaction requires the self-consistency in the variational procedure [6]. We introduce an auxiliary (non-observable) antikaon “binding energy” B_K to control the energy \sqrt{s} of the $\bar{K}N$ subsystem within the K^-pp cluster. This B_K is defined as

$$-B_K \equiv \langle \Psi | \hat{H} | \Psi \rangle - \langle \Psi | \hat{H}_N | \Psi \rangle ,$$

where \hat{H}_N is the nucleonic part of the Hamiltonian. The relation between the $\bar{K}N$ two-body energy \sqrt{s} and B_K within the three-body system is not *a priori* fixed. In general, \sqrt{s} can take values $M_N + m_K - \eta B_K$, where η is a parameter describing the balance of the antikaon energy between the two nucleons of the $\bar{K}NN$ three-body system. One expects $1/2 \leq \eta \leq 1$. The upper limit ($\eta = 1$) corresponds to the case in which the antikaon field collectively surrounds the two nucleons, a situation encountered in the limit of static (infinitely heavy) nucleon sources. In the lower limit ($\eta = 1/2$) the antikaon energy is split symmetrically half-and-half between the two nucleons. We investigate both cases and label them “Type I” and “Type II”, respectively:

$$\begin{aligned} \text{Type I :} \quad & \sqrt{s} = M_N + m_K - B_K , \\ \text{Type II :} \quad & \sqrt{s} = M_N + m_K - B_K/2 . \end{aligned}$$

Our calculation is then carried out such that self-consistency for \sqrt{s} is achieved, namely, the \sqrt{s} used in the effective $\bar{K}N$ potential is made to coincide with the \sqrt{s} evaluated with the finally obtained wave function.

Due to the elimination of the $\pi\Sigma$ and $\pi\Lambda$ channels, the effective $\bar{K}N$ potential is complex. We perform the variational calculation with the real part of the potential to obtain the wave function. The decay width is then calculated perturbatively by taking the expectation value of the imaginary part of the potential: $\Gamma_M = -2 \langle \Psi | \text{Im} \hat{V}_{\bar{K}N} | \Psi \rangle$, which represents the mesonic decay channels ($\bar{K}NN \rightarrow \pi\Sigma N, \pi\Lambda N$). The dispersive effect induced by the imaginary part of the potential and the non-mesonic absorption width for $\bar{K}NN \rightarrow \Sigma N, \Lambda N$ are treated separately.

3 Results

3.1 Structure of the K^-pp system

Here we present the results of the variational calculation. For an estimate of theoretical uncertainties, we have used four effective $\bar{K}N$ potentials derived from different versions of chiral models, and employed both the Type I and the Type II ansatz, as described in Ref. [7]. In all cases the K^-pp system turns out to be rather weakly bound as compared to previous calculations. As shown in the left panel of Fig. 1, the total binding energies range from 17 MeV to 23 MeV, and the mesonic decay width Γ_M ($\bar{K}NN \rightarrow \pi Y N$) lies between 40 and 70 MeV. The different versions of the chiral models give similar results within a relatively small window of uncertainties, while Type II ansatz gives slightly deeper binding than Type I by a few MeV. The reason for the shallow binding is found in the relatively weak $\bar{K}N$ potentials based on chiral dynamics. The chiral low energy theorem in the SU(3) meson-baryon sector dictates strong $\pi\Sigma$ attraction, and the coupled-channel dynamics locates the resonance structure in the $\bar{K}N$ amplitude at 1420

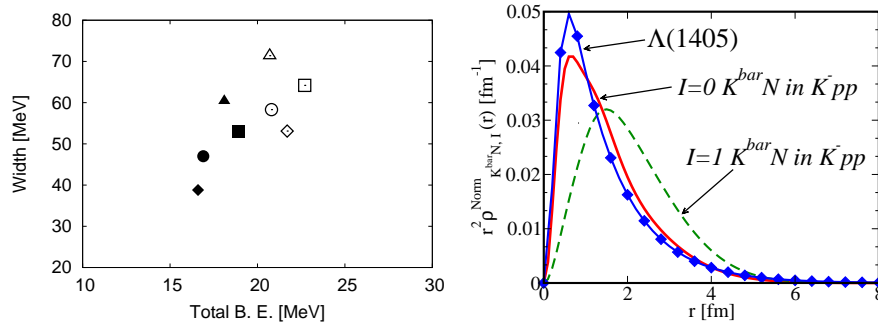


Fig. 1 (Left panel) Distribution of total binding energy and mesonic decay width. The results of four models are shown with different symbols. Closed (Open) symbols indicate the Type I (Type II) ansatz. (Right panel) Normalized and isospin-separated $\bar{K}N$ relative density in \bar{K}^-pp for a chiral model [10] with the Type I ansatz. Solid (dashed) line shows $I=0$ ($I=1$) $\bar{K}N$ density. Solid line with diamond shows the $\bar{K}N$ density of $\Lambda(1405)$ in the same model. All densities are displayed with r^2 -multiplied.

Table 1 Detail of the result with a chiral model [10] and Type I ansatz. “B. E. (\bar{K}^-pp)” and “ T_M ” are total binding energy and mesonic decay width of \bar{K}^-pp . “ R_{NN} ” (“ $R_{\bar{K}N}$ ”) is the NN ($\bar{K}N$) relative distance. “ $R_{I=0(1)}^{\bar{K}N}$ ” is the $I=0$ ($I=1$) $\bar{K}N$ relative distance. “B. E. (Λ^*)” and “ $R_{\bar{K}N}(\Lambda^*)$ ” are the binding energy and $\bar{K}N$ mean distance of isolated $I=0$ $\bar{K}N$ system. Energies and width are given in unit of MeV, while distances are given in fm.

B. E. (\bar{K}^-pp)	T_M	R_{NN}	$R_{\bar{K}N}$	$R_{I=0}^{\bar{K}N}$	$R_{I=1}^{\bar{K}N}$	B. E. (Λ^*)	$R_{\bar{K}N}(\Lambda^*)$
16.9	47.0	2.21	1.97	1.82	2.33	11.5	1.86

MeV, displaced from the 1405 MeV measured in the $\pi\Sigma$ spectrum. The binding energy of the isolated $\bar{K}N(I=0)$ system is about 12 MeV measured from $\bar{K}N$ threshold.

Table 1 shows a typical result of \bar{K}^-pp calculated with a chiral model [10] and the Type I ansatz. The results of the other cases under study are essentially the same. The mean distance between two nucleons, R_{NN} , is about 2.2 fm which is smaller than that of the deuteron (about 4 fm) and close to the NN distance in normal nuclei, but the system is obviously not much compressed.

It is interesting to compare the $\bar{K}N(I=0)$ component in \bar{K}^-pp with the $\Lambda(1405)$ as the $\bar{K}N(I=0)$ two-body quasibound state. The mean distance of the $\bar{K}N$ pair in \bar{K}^-pp is found to be close to that for $\Lambda(1405)$, namely $R_{I=0}^{\bar{K}N} \simeq 1.8$ fm and $R_{\bar{K}N}(\Lambda^*) \simeq 1.9$ fm. Calculating the expectation value of the relative $\bar{K}N$ orbital angular momentum, it turns out that the $\bar{K}N(I=0)$ pair is dominated by s -wave, just as the $\bar{K}N$ pair forming the $\Lambda(1405)$. The structure of the $\bar{K}N(I=0)$ pair in the \bar{K}^-pp system is thus similar to that of the $\Lambda(1405)$. The right panel in Fig. 1 shows the $\bar{K}N$ relative density distribution of $\bar{K}N(I=0$ and 1) components extracted from \bar{K}^-pp which are normalized to compare with that of $\Lambda(1405)$. Apparently, the distribution of $\bar{K}N(I=0)$ pair in \bar{K}^-pp is very similar to that of the $\bar{K}N$ two-body quasibound state.

3.2 Estimate of additional effects

Based on the wavefunction obtained above, we estimate the following contributions to the results which have not been taken into account so far: 1) dispersive corrections by the imaginary part of the potential, 2) effect of the p -wave $\bar{K}N$ interaction, and 3) decay width from the two-nucleon absorption process [7].

First, we consider the dispersive correction induced by the imaginary part of the $\bar{K}N$ potential. This effect can be calculated explicitly for the two-body $\bar{K}N$ system, by comparing the bound state solution of the real part of the potential with the resonance structure observed in the scattering amplitude with original complex potential. Examining four chiral models, we find an attractive shift of the binding energy 6 ± 3 MeV in the two-body $\bar{K}N$ system. We therefore estimate that the dispersive correction would add another $\Delta B \lesssim 15$ MeV to the binding energy of the K^-pp system.

Secondly, the contribution of the p -wave $\bar{K}N$ interaction is estimated perturbatively with the p -wave $\bar{K}N$ interaction:

$$v_{\bar{K}N}^{p\text{-wave}}(\mathbf{r}_{\bar{K}N}, \sqrt{s}) = V_{\bar{K}N,p}^0(\sqrt{s}) \nabla \exp[-\mathbf{r}_{\bar{K}N}^2/a_p^2] \nabla. \quad (3)$$

The coefficient $V_{\bar{K}N,p}^0(\sqrt{s})$ is complex and a detailed expression is given in Ref. [6]. A prominent feature in the p -wave interaction is the $\Sigma(1385)$ resonance below the threshold. Since the K^-pp system is weakly bound and the energy variable \sqrt{s} lies slightly above the $\Sigma(1385)$ resonance, the p -wave contribution to the binding energy is repulsive, about -3 MeV. The decay width is increased by $10 \sim 35$ MeV, because of the large imaginary part around the $\Sigma(1385)$ resonance structure.

Next, we estimate the contribution of the two-nucleon absorption process (non-mesonic decay width, $K^-pp \rightarrow YN$). The width is calculated with the correlated three-body density $\rho^{(3)}(\mathbf{r}_K, \mathbf{r}_1, \mathbf{r}_2)$ as

$$\Delta\Gamma_{abs}(K^-pp \rightarrow YN) = \frac{2\pi B_0}{\omega} \beta_{pp}(\omega) \times \int d^3\mathbf{r} \int d^3\mathbf{x} \rho^{(3)}(\mathbf{r}, \mathbf{r}, \mathbf{x}) G(\mathbf{x} - \mathbf{r}; a).$$

This is a generalization of the formula for K^- absorption on proton pairs in a heavy nucleus, where the coupling constant is constrained by a global fit to the kaonic atom data [11]. For the application to the few-body system, we modify the delta function type interaction to the finite range Gaussian form. This procedure is necessary to account for short range correlations of the nucleons in the few-body system, and physically motivated by the underlying mechanism of the meson-exchange picture. Using the correlation density obtained from the wave function of the K^-pp , the two nucleon absorption width is estimated to be 4 - 12 MeV.

4 Summary and discussion

We have investigated the K^-pp system with a variational method, employing a realistic NN potential (Av18 potential) and an effective $\bar{K}N$ potential based on chiral SU(3) dynamics. With theoretical uncertainties in the model, the binding energy and decay width of the K^-pp turns out to be

$$B.E.(K^-pp) = 20 \pm 3 \text{ MeV}, \quad \Gamma_M(K^-pp \rightarrow \pi YN) = 40 - 70 \text{ MeV}.$$

As a consequence of the strong $\pi\Sigma$ interaction in chiral scheme, the strength of the $\bar{K}N$ interaction is reduced and therefore we find a weakly bound state. The $\bar{K}N(I=0)$

pair in the obtained wave function of the K^-pp system exhibits a similar structure as the $\bar{K}N$ two-body quasibound state in vacuum. We have estimated corrections, such as dispersive correction, the p -wave $\bar{K}N$ potential, and the two-nucleon absorption process. Taking these effects into account, the total binding energy increases slightly and the total decay width becomes as large as 60 - 120 MeV.

Our result should be compared with another three-body Faddeev calculation with chiral interaction [4], where the K^-pp state was found with 80 MeV binding energy. While Faddeev approach treats the coupled channels explicitly, our variational calculation works by eliminating the $\pi\Sigma N$ channel. Although the two-body $\pi\Sigma$ dynamics is fully incorporated in the effective $\bar{K}N$ interaction, the dynamics of $\pi\Sigma N$ three-body system may generate additional attraction (see also Ref. [12]). In the coupled-channel framework, the obtained state is the mixture of the $\bar{K}NN$ and $\pi\Sigma N$ components, as the $\Lambda(1405)$ resonance in $\bar{K}N$ - $\pi\Sigma$ system. In this sense, our strategy is to focus on the $\bar{K}NN$ component, and the present framework may not be sensitive to the $\pi\Sigma N$ component.

Based on a recent experimental analysis, a broad structure at about 100 MeV below the $\bar{K}NN$ threshold is reported [13], the maximum of which coincides with the $\pi\Sigma N$ threshold. In the chiral framework, such a broad state in the deep subthreshold region would be interpreted in terms of $\pi\Sigma N$ dynamics, driven by the strong $\pi\Sigma$ attraction. The present investigation is however not capable to deal with the $\pi\Sigma N$ component, since we have eliminated this channel. While the new report [13] is an interesting observation, more careful analysis is needed to answer the question about its detailed structure.

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